The Effect of Income Taxation on Reference Points and Risk Aversion

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Abstract

In this paper, we consider the experiments that have been conducted to date on the effects of income taxes on the willingness of investors to take on risk. Interestingly, different experiments yield different conclusions about the effect. Some indicate income taxes increase risk-taking, while others indicate they reduce risk-taking. This paper attempts to give a coherent view of these experiments. In particular, we incorporate the results of a number of decision theory models by arguing that the key difference in these experiments is the effect of taxes on the reference points of the subjects. The paper discusses how reference points are affected by the structure of the choice environment in each of the experiments. The paper then discusses what lessons this may give for the structure of taxes in the far less controlled environment of actual investment markets.

Keywords: decision-making, investment, risk, taxation, utility

1 Introduction

The effect of taxes on the risk-taking behavior of investors has been a staple of the economic literature for many decades. (Sandmo, 1985) The nature of the relationship between taxes and risk-taking is foundational to tax policy as the long-term growth of an economy is considered to be greatly affected by the willingness of investors to take risks. (Obstfeld, 1994)

The standard model of investor behavior in the presence of income taxes was introduced by Domar and Musgrave in 1944. This model has been generalized and extended by a number of authors (e.g., Stiglitz (1969)) and has become an important pillar of the income tax literature. In its simplest form, the model predicts that investors will react to an income tax by shifting their investments in such a way as to obtain the same returns that they would have had without the tax. Thus, under standard assumptions of stability of preferences, etc., investors will shift their allocations to obtain the same net returns as they would have without any tax. Although the model is robust to a large class of utility functions,[1] it has a number of assumptions that make it difficult to test using archival data due to a large number of confounding effects in any real-world tax law changes.

[1]All classes of utility functions where the argument is a polynomial function of wealth provide a similar outcome as that suggested by Domar and Musgrave (1944), see Chorvat, Chorvat and Ekins (2018).

To understand the intuition behind the operation of the Domar-Musgrave model, the natural place to begin is to consider what sometimes referred to as a normative income tax, that is one in the which the tax is simply the rate times the income earned in the period. Under such a tax, if the net return on an investment is r and the tax rate is τ, then the tax on the return is τr, whether r is positive or negative. The symmetric treatment of gains and losses, sometimes referred to as allowing for a full loss offset, is the key to conclusions of the model. Investors are presumed to attempt to maximize after-tax returns. Here, the after-tax return on the investment is $(1-\tau)r$ whether r is positive or negative. That is under this tax, both the upside and downside of the investment are reduced proportionately by a factor of $(1-\tau)$. The effect of the full loss offset is to reduce the variance of any investment in that both the gains and losses are reduced by tax to the same extent. This reduction in variance is exactly proportionate to the reduction in the positive expected return.[2] Therefore, investors can mitigate reduced expected returns due to taxes by increasing the portion of their portfolio allocated to risky assets without a reduction in utility.[3] This is possible because the investor can cover the increased tax burden through assuming more pre-tax risk than would otherwise be acceptable to that individual without the tax,[4] even though the after-tax risk and return remains the same. Thus, if an investor can have the same risk profile and obtain the same returns after the imposition of a tax by scaling up the investment in the risky asset by a factor of $1/(1-\tau)$.

These predictions can be generalized to assets with different tax rates. Agnar Sandmo (1989) has shown that if there are two risky assets whose returns are independent and identically distributed, and the incomes of each are taxed at two different rates, then the more highly taxed asset will have a greater allocation of the portfolio than the lower taxed asset. These issues are discussed more fully in Weisbach (2004).

There are a number of challenges that arise in attempting to test this model empirically. First of all, real tax systems are much more complicated than what is described in this model and these complications may interact unpredictably with variables within the model (see Ch. 2, Bernard Salanie, The Economics of Taxation (2003)). As a result, it is difficult to obtain any meaningful conclusions from natural experiments.[5] These complications can be reduced by using a laboratory experiment in which nuisance variables can be controlled. This methodology provides a more direct test of Domar-Musgrave model.

This last point might seem to introduce a threshold question: If the model appears to be non-testable, then why try to test it? That is, if we cannot use it to predict real-world behavior directly, then why do we care if it is accurate? The answer to this derives from the idea that economic research should proceed in a manner similar to that of most physical sciences: that is, we should attempt to understand choices in simple situations (often unrealistically simple) and then introduce complications which make the analysis more realistic. Rather that attempting to understand investor behavior in all of its complexity at once, one should attempt to reduce it to simple choices and then add in complexity to more closely approximate real decisions. (Smith 1982) However, because what are often considered to be simple decisions involve complex behavior, even simple behavior needs to be investigated from many angles.

[2] This assumes that the expected return of the investment is non-zero. If the expected return of the investment is zero (e.g., a fair bet), then after this tax, the expected return will remain zero.

[3] One might be concerned that we are necessarily assuming something like quadratic utility (as we are here only discussing mean and variance). The discussion here is intended to give the reader the intuition behind the model. In fact, the conclusions are far more general than that as shown in Chorvat, Chorvat, and Ekins (2018).

[4] One should note that these models assume that the riskless rate of return is zero. If the riskless rate of return is greater than zero, the predictions of the model become ambiguous as the tax on these riskless returns cannot be avoided by scaling up investment. See Stiglitz (1969).

[5] For a survey of the attempts to test this hypothesis and the results obtained see James Poterba "Taxation, Risk-Taking and Household Portfolio Behavior" in The Handbook of Public Economics (2002).

The usefulness of experiments comes from the hope that the heuristics that individuals use in "regular" decision making is applied to the situations which confront the subjects within the experiment. That is, if the decision process used by the subjects in the experiments is sui generis and does not apply outside of this particular experiment, the usefulness of the experiment is rather minor. However, if the results of experiments elucidates the manner of decision-making used in a broader context of economic choices made by individuals, then it give us some insight about the effects of actual tax systems. In particular by examining the results of a number of experiments, we hope to discern how income taxes alter the choices made concerning risky investments.

Another point to consider is that when the tax imposed is progressive or does not allow for full loss offsets, the predictions become unclear. These two potential features of a tax law are connected in that if the income tax is progressive, it may be that gains will be taxed at a higher rate than the tax benefit that losses will give. Effectively, the income and substitution effects are in conflict and the resolution will depend on the assumptions about the utility function (Stiglitz 1969).

One threshold point we believe is worth noting: there is no objectively correct level of risk aversion, Consequently, there is no objectively correct answer to a portfolio allocation between a risky and a riskless asset. In most economic models of investment, risk aversion is simply taken as a primitive, and assumed to be stable. The attempt of this paper is to interpret the research to date on the effect of taxes on risk-taking in the context of other work on decision theory especially that which relates to behavior in experiments.

One possible criticism of the application of the experiments to taxes extent in the world is that in general these experiments only consider the choice between two different assets, while in the world there is a large choice among potential investments. However, under some version of standard financial theory, investors real choices are between a risky, but diversified, portfolio that is essentially the market portfolio and a riskless asset. This is sometimes referred to as the two-fund separation theorem and is a result of CAPM (Back, 2015). Thus, this form of the investment problem is not necessarily unrealistic.

We know that many things influence the level of risk aversion. (e.g, time of day, day of week, gender, hunger, etc.) (Diaz and Esparcia, 2019) Also, events that occur within the experiment such as having just won or lost a gamble, also seem to influence the willingness to take on risk. (Chorvat, Chorvat and Ekins, 2018). Next we review the experiments that have been conducted so far that directly focus on the effect of income taxes on investment risk-taking and their results.

2 Experiments On Income Tax and Risk-Taking to Date

Several studies have used laboratory experiments to determine the effects of capital gains taxation on investor behavior. The literature on the effects of taxation on investment is rather large. In this paper, we are focused on the experiments which directly study the effects of taxation on risk-taking. One should note that these experiments are considering decisions made by investors, rather than entrepreneurs or business owners. That is, the returns earned by the investments in these experiments are exogeneous to the choices made by the individual subjects. The subjects may choose which investments to make, but they do not control the underlying returns on those investments.

2.1 Swenson (1989)

This is really the first experiment to look at the effects of income taxation on risk-taking. This experiment looked at a market equilibrium situation rather than the partial equilibrium that was analyzed by

Domar and Musgrave. This experiment was comprised of buyers and sellers and used a double auction mechanism to explore the pricing of risky assets in the presence of a capital gains tax. This study used three different treatments: two of the treatments involved the use of a proportional tax of 10 percent and 40 percent respectively, and a third treatment used a progressive tax (i.e., one where the tax rate increased as income increased). This experiment found that buyers allocated 36.8 percent of their portfolio to risky assets in the untaxed condition and 38 percent in the 40 percent tax condition. This slight increase is certainly not consistent with the Domar-Musgrave hypothesis[6] nor was it statistically significant. In a very real sense though this experiment was not directly testing the model because this was utilizing a market equilibrium mechanism, where decisions made by investors can affect prices and thus the returns on their investments. That is, if investors wish to allocate more of their portfolios to risky investments, the price of those investment would rise, thus reducing the returns. The lowering of the returns on the risky investments might then decrease the desire of investment to allocate more capital to risky investments. In this experiment, there was no statistical difference in either portfolio allocation or price of securities between the proportional tax and the untaxed condition. Note that because they did not scale-up their investments in the risky assets, they effectively behaved as if their risk aversion had increased. In this experiment, there was no statistical difference in either portfolio allocation or price of securities between the proportional tax and the untaxed condition. Thus, it appears that the investors did not increase their demand for taxed assets. Note that because they did not scale-up their investments in the risky assets, they effectively behaved as if their risk aversion had increased.

The subjects did behave differently in the condition where there was a progressive tax under which gains might be taxed at a higher rate than the tax benefit given to losses. In this condition, the prices of the risky asset were lower than in the other conditions and the average portfolio allocation to the risky asset dropped to 24.9 percent. However, because of this mechanism, we cannot tell what the subjects would have done in a simple portfolio allocation problem.

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2.1.1 King and Wallin (1990)

This experiment used a simplified portfolio task to test if subjects would increase their allocations to risky investments under a proportional tax with offsets as compared to an untaxed condition. The experiment also in involved a separate condition in which a progressive tax was imposed and which allowed for loss offsets, however the tax benefits on losses were no progressive, thus resulting in potential asymmetries in the tax rates. This study showed an initial increase of risky investments in both tax treatments, but after the first few periods, there was no difference between the taxed and untaxed treatments in allocation of the portfolio to the risky investment in the proportional tax conditions. After the imposition of a proportional tax, investors chose a lower expected return and lower risk than they did without the tax. Thus, contrary to the conclusions of the Domar-Musgrave model, the imposition of a proportional tax resulted in a change in the net returns of the subjects. The subjects also decreased their allocation to the risky asset in the progressive tax conditions as well.

2.1.2 Meade (1995)

In this experiment, the subjects were given tokens that were redeemable after the end of the experiment. There were two periods in the each round of the experiment. In the first round subjects chose between

[6] Had they behaved consistently with the model, they would have increased their allocations to the risky asset by 1/(1-.4) or 66 percent, thus they would have shifted from an allocation of 36.8 percent to the risky asset to an allocation of 61 percent.

redeeming the tokens in period 1 or investing them. If they invested them, they would receive the results of the investments in period 2. If the subjects chose to redeem the tokens in the first round, they could begin to use them for consumption at local businesses in 30 days immediately following the experiment. If they chose to invest the tokens, then whatever the value of the invested tokens was after the returns on the investments was determined could be used in the 30 days immediately following a day three months after the experiment. The tokens that they decided to invest were then allocated between a riskless investment or a risky investment. For some of the subjects would be a 25 percent tax on any gains made during the experiment, while for others gain was not taxed. They found that mean risk taking in the no-tax environment was 59.02 and that in the taxed condition the amount of risk-taking increased to 74.46. Had they made the Domar-Musgrave shifts the amount of risk-taking would have increased to 78.7, which is not terribly far from what did occur. The standard errors for these were easily sufficient to include the Domar-Musgrave shifts. So, one can conclude that this experiment is consistent with this prediction.

There were also conditions in the experiment in which the returns would be taxed, but at an uncertain rate. In these conditions, the subjects allocated less to the risky asset than they had in the certain environment, but more than in the untaxed environment. One interesting point to note about the subjects to this experiment is that they were selected from a larger pool of volunteers based on the answers to a questionnaire. The subjects that seemed to exhibit constant relative risk version were selected for the experiment.

2.1.3 Fochmann, Kiesewetter, and Sadreieh (2012)

This experiment focused on different methods of accounting for losses. This is the first of the papers discussed which used an interesting mechanism to test the effect of taxes on risk-taking. In these experiments, the nominal returns on the risky choice were grossed up for whatever tax was imposed. E.g., if there was a 40 percent proportional income tax imposed, then the nominal returns on the risky asset would be 1/(1-.4) or 2/3 larger than they would be in the untaxed condition. Thus, in some sense it makes the comparison easy between the taxed and untaxed rounds. Under this mechanism the Domar-Musgrave predicted shifts in portfolio allocation have effectively already been made for the investors. If subjects maintain the same portfolio allocation as before, they will keep the same risk-return ratio as in the untaxed condition. Any change in the allocation changes the risk and the returns the subject is receiveing. Of course, this means that if the after-tax returns were the key focus of the subjects in determining their choices, the allocations made should be constant across conditions. However, to the extent that there were differences across conditions (and there generally were), one cannot be sure if the changes made were due to the tax differences, or due to the differences in nominal returns. That is, while as part of the experiment the subjects were made aware of this, it may still be possible that the changes to the nominal returns affected choices (even though in some sense they should). Therefore, because more one thing was changed, across the different conditions, we cannot be sure what actually affected the choices.

In this particular experiment, the subjects first "earned" their endowments by performing a task. The subjects then made a choice of how to allocate their endowment between two investment funds. One might note that both investment funds had the same expected value, but one had a larger variance (called the risky fund) than the other (called the safe fund).[7] The focus of the experiment was the effect of differing rules to offset losses. They tested four conditions: Baseline (no tax), Tax Control (gains are taxed, but there was no tax benefit from losses), Partial loss offsets and Capped loss offsets. They found that there was a reduction in risk-taking from the no-tax to the Tax Control condition, although not

[7] Of course, if the only difference between the two investments is the variance, a risk-averse investor should always invest her entire portfolio in the "safe" asset.

very large. They found that there was an increase in risk-taking when either partial or capped loss offsets were allowed, even relative to the no-tax condition. It might be worthwhile to note that this is not the paradigmatic Domar-Musgrave situation in that this is not really a portfolio allocation, but a choice between two lotteries. In addition, there was no proportional tax condition.

2.1.4 Ackermann, Fochmann, Mihm (2013)

This experiment also used the gross-up mechanism discussed above. Thus, any differences in portfolio allocation are not due to a concern with after-tax returns, but are the result of some other facet of the choice environment. In one condition of the experiment, a 50 percent tax was imposed on the returns on the risky asset, while in another, a 50 percent subsidy was awarded. In both of these conditions, the average percentage of the portfolio allocated to the risky asset by the subjects was lower than that in was in the untaxed condition. Thus, both the tax and subsidy resulted in an apparent lowering in the willingness to tax on risk. As they simplified the situation the reduction in risk-taking was decreased but still present. That is, as they made the payoff structure of the risky asset simpler, the subjects were willing to take on more risk, however even in the simplest structure the willingness to take on risk was less than it had been in the untaxed condition.

2.1.5 Falsetta, Rupert and Wright (2013)

This experiment looked at the effect of the timing of changes to the capital gains rate. In this experiment, losses were fully deductible, so it comports with the assumptions of the Domar-Musgrave model. In this experiment, imposing a higher tax actually slightly reduced the allocation to the risky asset, and decreasing the tax increased the amount allocated to the risky asset. This is the opposite of what would be predicted in the Domar-Musgrave model. The experiment also had conditions where the tax increases or decreases were imposed in phases as opposed to all at once. They found that gradual increases or decreases had a larger effect than immediate ones (although the directions did not change).

2.1.6 Fochmann, Hemmerich and Kiesewetter (2016)

In this experiment, subjects were given an endowment and they were askd to choose the amount of their portfolio to allocate between risky and riskless assets. There was an untaxed condition and one in which gains were subject to 50 percent tax. This experiment used the gross-up procedure discussed above, so again if net returns were the goal, choices should have been the same in all conditions. In addition to the untaxed and taxed conditions, there were differences in the amount of information provided about the returns given to the subjects, with low, medium and high information conditions. That is, in the high information conditions the subjects were shown explicitly what would happen in each of the two states for any proposed allocation. In each information condition, subjects were more willing to invest in the risky asset in the taxed condition than in the untaxed condition, although interestingly this difference was highest in the low information condition (about a 5.5 percentage point difference) versus the high information condition (about a 2.5 percentage point difference).

The experimenters here also surveyed the subjects about the perceptions of the experiment along three dimensions. Here they found that the only question which was statistically significantly related to portfolio allocations was what they called Perception. This question asked how risky the subject viewed the investment in the taxed condition. The answer to this question was positively and statistically significantly correlated to the shifts in the allocation to the risky asset.

2.1.7 Fochmann, Hewig, Kiesewetter, Schussler (2017)

In this experiment, the subjects could choose between a low-risk lottery and a high-risk lottery. This experiment also used nominal returns were grossed up for the tax effect, so that the choices based on net returns would be the same across all conditions. There were two different treatment conditions, first a tax on gains and second, a reduction in losses. They found that the tax on gains reduced the number of subjects who chose the high-risk lottery, while the ability to recover some of the loses increased the likelihood of choosing the risky lottery.

The experimenters here also surveyed the subjects about the perceptions of the experiment along three dimensions. Here they found that the only question which was statistically significantly related to portfolio allocations was what they called Valence. This question asked on a scale of 1-10 how positively the subject perceived the tax. They found that the answer to this question was positively and statistically significantly related to the increase in the allocation to the risky asset.

2.1.8 Fochmann and Hemerich (2018)

This experiment involved a portfolio allocation task. Again, the nominal returns were grossed up, so that the choices based on net returns would be the same across all conditions. This experiment found that there was a significant decrease in the willingness of individuals to take on risk when a proportional tax is introduced. This means that under the tax conditions, the subjects exhibited greater risk aversion that in the untaxed conditions.

2.1.9 Chorvat, Chorvat, and Ekins (2018)

This experiment tested the Domar-Musgrave hypothesis in its paradigmatic setting. That is, the returns were not grossed up. The tax structure here allowed for a full loss offset. Furthermore, in this experiment, the subjects were allowed to see what would have happened from their allocation at the end of each round (although their eventual payoff was based on only way randomly selected round). In this experiment, the subjects did not have a significant reaction to the imposition of symmetric capital taxes in the amount of risk they were willing to undertake. This experiment found that the most important determinant of the level of risk-taking in any given round was the return on the risky asset in the immediately prior period.

The results of this experiment are consistent with those that have found that investors tend to "chase returns" in investments such as mutual funds (Delong, Shleifer, Summers and Waldmann 1990). If investors are "chasing returns," this behavior may outweigh the effect of taxes on risk-taking. This is the only one of the experiments discussed to consider the relation between prior asset payoffs and subsequent portfolio allocations.

3 Results

Looking at these experiments, one can see that the results vary significantly across the experiments. Some of the experiments show that income taxes increased willingness to invest in risky assets and others showed that it decreased it. One should remember that the Domar-Musgrave argument showed was that in the case of full loss offsets, an investor can obtain the same returns and as risk as without a tax, if the increase the allocation to the risky asset by a proportion of $1/(1-\tau)$. Thus, for those experiments which did not use the gross-up mechanism, if there was not an increase in the allocations to risky asset then, in fact, the subjects were decreasing their risk-taking. For experiments that did use the gross-up mechanism, the the answer was clearer as to whether they were increasing or decreasing

the riskiness of their portfolios.

What almost all of these experiments show is that taxes introduce intransitive choices by the subjects. That is, even when the same choices are given to the subjects, but the context is changed by the introduction of a tax, the subjects make different choices. That is, if we assume that risk preferences are stable, then something about the choice environment in the taxed conditions altered the subjects' choices. This is particularly clear in the experiments that used the gross-up mechanism. In essentially all of these experiments, the subjects made different choices about their after-tax returns than in the returns of in an untaxed condition. Thus, one requirement of any explanation of these experiments is that it must explain the intransitivity of the choices made in the experiments.

One exception to this intransitivity is Experiment 3, or Meade (1995). This experiment can be seen as agreeing with the Domar-Musgrave model. There is a concern in applying the results of this experiment more broadly though. First, the subjects were chosen to the degree their choices conformed to standard utility theories. Second, the payoffs of the investments would not occur for over three months after the experiment. It may be that this length of time is enough to detach the choices from reference point analysis below.

One way to make sense of these intransitive behaviors is utilize the idea that individuals base their decisions not on their total wealth, but rather on a reference point. The idea that individuals make their decisions based on reference points, as opposed to total wealth goes back at least to Friedman and Savage (1948) and Markowitz (1952). It was given a new degree of attention following the publication of Kahneman and Tversky (1979, 1992) which introduced prospect theory and cumulative prospect theory respectively. There are a number of other features of these models, but the one we focus on in this paper is the reference points.

One of the reasons for focusing on the reference point, as opposed to other characteristics of the decision-making process, is that this is likely the one that is most affected by the set-up of an experiment. For each of the subjects of any given experiment characteristics such as risk-aversion, total wealth, etc. are very likely going to be the same in all of the conditions of an experiment. However, what the subject views as a good or a bad result will very likely be determined by the choice environment given by the set-up of the experiment.

Reference points are generally considered to be difficult to model (Wakker, 2015). That is, they are a creature of what aspects of the choice environment the subjects focus. What aspects of an even mildly complicated choice environment are generally difficult to determine. However, as is apparent from the decision theory literature, reference points are central to understanding decisions made in experiments. So while it is difficult to model how reference points are formed, it is crucial.

The idea of reference points has also been used in order to explain a number of real-world phenomena which might otherwise be anomalous such as the equity premium puzzle among others (Constantinides 1981, Campbell and Cochrane 1999). In these models, the current consumption levels are used as the reference point from which decisions are made.

4 Endogenous Reference Points

To the extent that taxes affect the choice environment in such a way that the reference point used by the subjects has been altered, this can change the decisions made by the subjects. That is, while Lottery

A may be preferred to Lottery B in one situation, in a different situation Lottery B will be preferred to Lottery A if the reference point is changed. Thus, transitivity can be violated which, as noted above, is required to explain the results of most of these experiments.[8]

At a high level of generality, changing the choices between two lotteries can be found in standard expected utility framework. The idea of relative risk aversion indicates that there are certain gambles that one might be willing to accept when one's wealth is \$10 million that one would not accept when one's wealth is \$100,000. This is not viewed, however, as a violation of expected utility transitory in this context, because in expected utility all choices are viewed as based on final wealth.[9] Because the final wealth choices are not the same when there is a large difference in beginning wealth, we would not have a violation of transitivity. Because, in the experiments discussed above, final wealth is essentially unchanged - given that the payoffs were small relative to total wealth - and so expected utility cannot explain the choices made in these experiments. Rabin (2000).

We argue that the differences observed in the experiments can be attributed to the experimental design an in particular the effect on the reference points of the subjects as the face the choice in the experiments. To illustrate the difference that a change in reference points can make, let us consider an individual who has preferences based on a fairly standard model, namely constant relative risk aversion (CRRA) with a reference point and loss aversion. This is also sometime referred to as shifted CRRA utility (Back, 2015) The utility function that would correspond to this model would be:

$$\mu(I) = \begin{cases} \frac{(I-\rho)^{\gamma}}{1-\gamma}, I > \rho \\ \\ -\lambda\frac{(\rho-I)^{\gamma}}{1-\gamma}, I < \rho \end{cases}$$

where ρ is the reference point, I would be income or wealth, and γ is a risk aversion parameter (in Arrow-Pratt terms, it is the coefficient of relative risk aversion) and λ is a loss aversion parameter. The second derivative of the losses is positive, so that we have risk aversion for gains and risk preferring for losses.

If we consider a situation in which an individual is confronted with a choice of either a certain gain of \$.5 or a 50 percent chance of a \$5 gain and a 50 percent chance of a \$4 loss, we can see what the individual would choose if they had parameter values of ρ=0, γ=2, and λ=-1.5.[10] In this case, the individual would find that the utility value of the certain gain of \$.5 would be greater than the utility of the gamble. If however, the reference pint were to be moved to anything greater than \$1.02, we would have that the utility of the gamble would exceed that of the certain amount.

The reason that this shift in the choice occurs is that the utility effect of losses is greater than the utility effect of gains. When the reference point shifts up, while the gamble's loss is treated as larger than before and the gain is treated as smaller than before, the certain gain is treated as a certain loss. The individual would now prefer to take the risk, rather than suffer the certain loss. Thus, we can see that if the reference point of an individual can be altered, the perception of the risk is changed and so too are the choices made by that individual.

[8] One could argue that in fact transitivity has not been violated because, if the reference point has been changed, the decision problem has been changed. Thus, one cannot really expect the decision to be the same in both situations.

[9] That is, the decision in case of \$100,000 ± x is not the same as \$10,000,000 ± x.

[10] These choices for parameters are merely for illustration, although they are not unreasonable given the empirical results in a broad number of experiments. Campbell and Cochrane (1999).

As noted by Markowitz (1952), it is difficult to model reference points as it seems like they a product of internal psychological process which might be quite heterogeneous. Wakker (2015) notes that the situation is not much better more than half a century later. There have been some attempts to endogenize reference points. The most prominent of these have been Schmidt (2003) and Koszegi and Rabin (2006). In these articles, the authors essentially argue that individuals update their reference points based on changes to their expectations about what will occur in the future. In both of these cases, a reference point can be stochastic. That is, the reference point might be the payoff of a particular investment, rather than a set number of dollars (although obviously, if the investment used as reference gives a certain return, the reference point will be the same in all states).

In their model, Koszegi and Rabin (2006) have two parts to utility. The first part is a standard utility that is determined by something like total wealth. They also have a second part which they refer to as win/loss utility. Thus the utility function takes on the form (using their notation):

$$u(c|p) = m(c) + n(c|p)$$

where m(c) is that standard utility of a consumption level c and $n(c|p)$ is the gain-loss utility. It is fairly clear that given, the size of the potential winnings in the experiments that we consider, the change to consumption and hence the change to m(c) will be fairly small and not likely to affect the decisions made as much as $n(c|p)$. Thus, we will focus on the structure of the gain-loss utility. In particular, we focus on the formation of the reference point ρ.

Kosezegi and Rabin (2006) hypothesize that reference points are based on the rational expectations from the choices that confront individuals. That is,

$$\rho = \int_{\infty}^{-\infty} F_i dQ(i)$$

Where F_i is the outcome in the ith state of a gamble or prospect that the individual faces and Q(l) is the probability measure of the possible outcomes of that gamble. This is to say that reference points are what individuals believes is the expected value of the outcomes that confront them. Below we discuss how this model of reference points does not seem to describe the effects of taxation on risk-taking in the experiments we discuss.

It seems intuitive that individuals come to the experiments with particular types of risk aversion and this is unlikely changed by the experiment (except in the cases where one uses an experimental procedure that effectively induces particular utility function).[11] It would seem that the experiment is more likely to affect the expectations. That is, the structure of the experiment is more likely to affect what might be thought of as a gain or loss. The work of Kahneman and Tversky indicates that the framing of the decision in an experiment may have a large influence on the formation of reference points and thus the choices made.

To say this another way, it would seem that reference points are likely significantly endogenous to the experiment, and any model of the decisions made in an experiment on taxes should at least to some extent consider the change in the reference points due to the imposition of taxes.

[11] See Hvide, Lee, and Odean (2019).

5 Modeling Reference Points

There is insufficient data in any of the experiments (or even really all of them combined) to make a determination of the detailed structure of the utility functions of the individual subjects. Indeed, the "true" utility functions of the subjects of the experiments are likely far more complicated than any of the standard models (the results of the experiments indicate this) and the functions are also likely heterogeneous. Thus, at best, any utility function which we use will be approximations.

One of the reasons why we are not giving much consideration to many of other aspects of prominent decision theory models such as cumulative prospect theory is that many of the features of these models are not likely be play a large role given the rather simple choice environments in most of these experiments. For example, a key part of cumulative prospect theory is the notion of probability weighting. However, given that for most the experiments the probability of both the high and the low payoff is 50 percent, probability weighting is not likely to play a large role. This is a point in many probability weighting schemes were the weights are equal to the value or at least where the distortions to probabilities are their lowest. Furthermore, since the experiments differ in their results, yet the probabilities are mostly all the same, this cannot be the explanation for the differences we observe.

Given our focus on reference points, we introduce our first proposition about the effect of changing the reference point on risk-taking.

Proposition 1: When the choice environment of an experiment increases (decreases) the reference point of the subject, it decreases (increases) the risk aversion of the choices made by the subject.

The proof of this proposition is in the Appendix.

We shall use this proposition to argue that the experiments which saw a decrease in risk-taking result from the choice environment decreasing the reference point and vice versa for those for which the subjects increased risk-taking in response to taxes. Let us now consider these experiments in detail.

For the majority of the experiments (i.e, experiments numbered 1,2,5,6,9, and 10 above), the introduction of a proportional tax reduced the willingness of the subjects to take on risk. One might break these experiments into two groups. In experiments 5,6 and 9, there was a real and clear shift to lower risk after the imposition of the tax. In three experiments (experiments 1, 2, 10), the subjects essentially did not change their portfolio allocation in nominal terms between the two conditions. While not changing the allocation was effectively reducing the risk of the portfolio, it is not initially clear if this would be better characterized as simply a lack of reaction to the taxes, or as a reduction of risk-taking. We should say that in each of these three experiments there was evidence that the individuals were willing to alter their allocations based on certain characteristics of the experiments. For example, in the experiment (10) above Chorvat et al. (2018) if the immediately prior pay had been positive, subjects were willing to invest more in the risky asset. Thus, one cannot attribute that lack of change in the portfolio allocation to mere conservatism. The subjects did seem to perceive the change in the net risk of the experiments. It would seem then that the subjects in these experiments viewed taxes as making them worse off, or shifting their reference points downwards. This then led to higher risk aversion and choosing lower risk portfolios.

In the experiment numbered 3 above, the subjects essentially had the same risk-return profile with the tax as they did without the tax. Thus, they behaved consistently with the predictions of the Domar-Musgrave model. There are some issues with the methods of this experiment is using to understand

behavior of investors generally. First of all the subjects were not randomly selected, but rather chosen on the basis of their use of something akin to constant relative risk aversion. Of course, if you believe that such a model is a good one for real investors, then this is not a significant problem. However, if you are either uncertain or this, or believe it to be incorrect, then the usefulness of the results of this experiment are diminished.

In experiments 4,7 and 8, the subjects seem to have actually increased their willingness to take on risk with the imposition of the tax. Interestingly in experiment 4, the imposition of an income tax without loss offsets reduced risk-taking, but when the ability to offset losses, albeit partially, was introduced, this resulted in an increased willingness to take on risk, even beyond that without a tax. Here one might argue that the reference points of the subjects were altered by the possibility of a tax without offset, so that allowing for any offset seemed like an improvement. This would increase the reference point and thereby increase the willingness of the subjects to take on risk.

In experiments 7 and 8, the experimenters asked the subjects about how they perceived the taxes in the context of the experiment. In both of these experiments, the subjects expressed positive views about the imposition of taxes. In experiment 7, the subjects indicated that they viewed the taxed condition as less risky than the untaxed condition and in experiment 8, they indicated that they had a positive feeling about the taxed condition as compared to the untaxed condition. This is interesting and gives some evidence for the idea that a more positive feeling can be modelled as an increase in the reference point.

From this evidence, we can conclude that these three experiments were different than the others. Something about the choice environment led the subjects in experiments 7 and 8 to view the taxes positively. It seems reasonable to think of this as a shift upwards in the reference point in the taxed condition in those experiments. For experiment 4, it would seem that by first introducing a tax with no ability to offset losses, the ability to offset losses is now seen as a positive, thus shifting the reference point upwards.

In some sense the experiments which utilized the gross-up mechanism seem to directly demonstrate that the model of the reference point developed by Koszegi and Rabin (2006) cannot not exactly correct. Recall that their model of the reference point was the rational expectations of the net results of the choices of the subjects. The mechanism used in these experiments using the gross-up mechanism made sure that the true value of the options available to subjects when taxed were exactly the same options they had when no tax was imposed, yet they in general made different choices. In such a case, the rational expectations would be unchanged between the taxed and the untaxed conditions. Thus the choices should be the same under the Koszegi and Rabin model. Thus, something other than merely the choices available must be affecting the formation of reference points.

6 Lessons for the Structure of Actual Income Taxes

The untested hypothesis is that these experiments are giving us some insight into the ways that investors consider taxes in their investment decisions. Note that the imposition of tax imposes a different choice environment in the real world. There are many complexities to investments and tax structure that do not exist in these experiments. For example, actual investments do not have simple payoff structures with known probabilities. Actual tax systems are more complicated and face uncertainty as to how the rules may change over time. That is because in addition to funding the consumption of the subject, this returns on the asset would also be funding something else, that is a public good.

This would seem to indicate that the way a tax is perceived might make a large impact on the effect of the tax. For example, does the tax make the individuals feel poorer (in some sense lowering the reference point) which would increase risk aversion and decrease the allocation to the risky asset or richer, which will increase the allocation to the risky asset. Given that the framing of a tax is likely outside of the control of the government, there is likely not a direct way in which these results can be used to craft tax policy. However, to the extent that we can determine particular tax structure which reduce the reference point as little as possible, or even possibly increase it, this could be used to minimize the effect of taxes.

The work by Falsetta et al. (2013) does seem to indicate that tax cuts should likely be phased-in, whereas tax increases should be implemented all at once, so as to maximize the effect of tax cuts and minimize the effect of tax increases. This is also consistent with the experiment Fochmann, Kiesewetter,and Sadreieh (2012) in which first having a condition disallowing the use of losses likely increased the positive effect on the reference points of conditions which only allowed for partial use of losses to generate tax benefits.

7 Conclusion

Clearly much more work needs to be done on the formation of reference points. In particular, it is important to gain a clearer understanding about what types of taxes have greater and lesser affects on reference points.

8 Acknowledgments

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9 Appendix

Proof of Proposition 1

This proof will give the results for two cases: 1. Linear utility with a kink at the reference point and 2. For standard assumptions about loss aversion, where $u"(x) \neq 0$. For both cases, we assume there is a risky asset that has two possible payoffs and a riskless asset. The higher payoff of the risk asset is greater than the return of the riskless asset and the lower payoff is below that of the riskless asset.

Case 1: Linear utility, with kink at the reference point.

In this case, the utility function can be written as:

$$u(x) = \begin{cases} \{(x-\rho), x > \rho \\ \\ \lambda(x-\rho), x < \rho \end{cases}$$

Where $\lambda > 1$. Under this utility function, losses can have more weight than gains of the same magnitude. If we choose a gamble where the individual who has this type of utility function is indifferent to

taking the gamble or not, we have that

$$pG + (1-\rho)L * G/|L| = 0$$

where G is the potential upside of the risky asset, which has a probability p of occurring and the lower return, L, has a probability (1-p) of occurring. In this case, the reference point $\rho = 0$, and then $G/|L| = \lambda$.

Now if we alter the reference point ρ so that it is no longer equal to zero, then we have that:

$$p(G-\rho) + (1-p)(L-\rho) * G/|L| = -p\rho - (1-p)\rho * G/|L| = -\rho 0.5((|L|p + (1-p)G)/|L| + G/|L|)$$

Versus now a sure loss of ρ, which in utility terms is

$$-G/|L|\rho$$

Note that, since $G>|L|$, or $G/|L|>1$,

$$\rho 0.5(1 + G/|L|) > \rho G/|L|$$

That is, if we set the gains and losses so that before the reference point moves, the subject is indifferent between taking the gamble and not taking the gamble. After the reference point has shifted, the subject will strictly prefer the gamble to not taking the gamble.

Case 2: Loss Aversion

Now we prove this for the cases in which we have a utility function that exhibits what is referred to as loss aversion. The basic idea here is that if we assume that subjects are risk averse as to gain and risk preferring as to losses, this gives us that from the definitions of loss aversion

$$u"(x) = \begin{cases} \{(u"(x) < 0), x > \rho \\ \\ \{(u"(x) > 0), x < \rho \end{cases}$$

We will also make the assumption that:

$$u(G) - u(G-\rho) < u(0) - u(-\rho)$$

If we also make the further assumption (as do Koszegi and Rabin 2007) that, if

$$y{>}x \geq 0, then$$

$$\mu(y) + \mu(-y){<}\mu(x) + \mu(-x)$$

.

Moreover, we assume that $G - \rho{>}\rho$ (that is, that the new adjusted value of the gain is viewed as a gain relative to the reference point).

These assumptions imply that:

$$u(G) - u(G - \rho) < u(0) - u(-\rho)$$

What we then have is that:

$$\left|\int_{-\rho}^{0} u'(x)dx\right| > \left|\int_{0}^{\rho} u'(x)dx\right| > \left|\int_{G-\rho}^{G} u'(x)dx\right|$$

The first inequality follows, translating the above equation into integrals, and the the last follows from the fact that:

$\upsilon''(x) < 0$ when $x > \rho$.

We then have on the loss side that, because the second derivative is increasing when $x < \rho$,

$$\left|\int_{-\rho}^{0} u'(x)dx\right| > \left|\int_{L-\rho}^{L} u'(x)dx\right|$$

We can then see that

$$p\left|\int_{G-\rho}^{G} u'(x)dx\right| + (1-p)\left|\int_{L-\rho}^{L} u'(x)dx\right| < \left|\int_{-\rho}^{0} u'(x)dx\right|$$

And thus the certainty equivalent will now be greater than zero, hence the level of risk aversion, or the willingness to take on risk, will have increased as a result of the shift of the reference point.

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